Multi-period decision making applied to cryptocurrency investment scenario

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Outline

Stylized facts of cryptocurrencies market

What do we know about cryptocurrencies?

- Mean and standard deviation are between 6 to 100 times larger than traditional assets
- Negligible correlation with standard assets such as stocks, bonds, and gold.
- Moderate correlation among cryptocurrencies, albeit time-varying.
- Long memory in returns and volatility.

Goal of the research

Study the suitability of dynamic linguistic information aggregation to support investment decision making.

The Efficient Market Hypothesis (EMH)

- Bachelier (1900) "Théorie de la spéculation" → first stochastic model in financial economics.
- Samuelson (1965) "Proof that Properly Anticipated Prices Fluctuate Randomly" → in a competitive market, price dynamics follow a random walk.
- Fama (1970) "Efficient Capital Markets: A Review of Theory and Empirical Work" → formal definition of EMH and classification.

Table: Descriptive statistics of daily returns. Coverage: 21/05/2014 to 27/09/2018.

	GOLD	S&P500	BOND	BTC	XRP	DASH	XRM
Observations	1134	1134	1134	1134	1134	1134	1134
Mean	-0.0079	0.0377	-0.0031	0.2221	0.4506	0.2351	0.3166
Std. Dev.	0.8104	0.7744	0.3216	4.5703	8.8985	7.9748	8.6688
Min	-3.2239	-4.1843	-1.5377	-26.4311	-57.0455	-73.3201	-36.6830
Max	4.6184	3.8291	1.3196	27.8435	136.3081	50.0787	69.1884
Skewness	0.2312	-0.5831	-0.0559	-0.1335	3.9416	-0.1229	1.3535
Kurtosis	5.6593	6.9085	4.0056	8.4510	57.5688	13.6400	12.0729

Source: Aslanidis, Bariviera & Martínez, *Fin. Res. Lett.*, **31**, 130–137 (2019)

Correlation among cryptocurrencies and selected traditional assets

Table: Correlation matrix.

	BTC	DASH	XMR	XRP	GOLD	S&P500	BOND
BTC	1.0000	0.2535	0.3161	0.1912	-0.0114	0.0356	-0.0371
DASH		1.0000	0.2072	0.2035	-0.0124	0.0190	-0.0574
XMR			1.0000	0.1639	0.0064	0.0004	-0.0699
XRP				1.0000	0.0171	0.0029	0.0411
GOLD					1.0000	-0.1023	0.2815
S&P500						1.0000	-0.3466
BOND							1.0000

Source: Aslanidis, Bariviera & Martínez,*Fin. Res. Lett.*,**31**, 130–137 (2019)

Our problem

Let consider a rational investor, who wants to invest in cryptocurrencies. She has to decide periodically (probably daily) to keep the cryptocurrency that had already bought or to sell it and buy another one.

Methodology

We adapt Xu (2014) Multi-period multiattribute decision making (MP-MADM) to the cryptocurrency market.

Zeshui Xu. On multi-period multi-attribute decision making. *Knowledge-Based Systems*, **21**(2):164–171, March 2008.

Xu (2014) formalizes the MP-MADM problem as follows:

- Let t be a time variable.
- Let t_p be P different time periods ($p = 1, 2, \ldots, P$).
- Let $\omega^t = (\omega^{t_1}, \omega^{t_2}, \ldots, \omega^{t_P})^T$ be the weight vectors of attributes through time, where $\omega^{t_p} \geq 0$, $\sum_{p=1}^{P} \omega^{t_p} = 1$.
- Let $\omega^{t_p} = (\omega_1^{t_p}, \omega_2^{t_p}, \ldots, \omega_m^{t_p})^T$ be the weight vectors of attributes at the period t_p, where $\omega_j^{t_p} \geq 0$ ($j = 1, 2, \ldots, m$) and $\sum_{j=1}^{m} \omega_j^{t_p} = 1$.
- Let $\mathbf{R}^{t_p} = (r_{ij}^{t_p})_{m \times n}$ be the linguistic decision matrices of p different periods, where $r_{ij}^{t_p}$ denotes the attribute assessment value provided by the decision maker over the alternative $\mathscr{A}_i$ with respect to the attribute G^j at the period t_p by using for instance, the additive linguistic evaluation scale S.
- Let $s_\alpha(t)$ be the additive linguistic label at the time t, where $s_\alpha(t) \in \overline{S}_2$ for any t, with $\overline{S}_2$ being the continuous linguistic evaluation scale.

Then, the dynamic linguistic weighted averaging ($DLWA$) operator is defined as

$$DLWA(s_\alpha(t_1), s_\alpha(t_2), \ldots, s_\alpha(t_p)) = \omega^{t_1} s_\alpha(t_1) \oplus \omega^{t_2} s_\alpha(t_2) \oplus \cdots \oplus \omega^{t_P} s_\alpha(t_P),$$

where $\oplus$ stands for the linguistic averaging operator

Let $f : [0,1] \rightarrow [0,1]$ be a Basic Unit-interval Monotonic (BUM) function (where $f(0) = 0, f(1) = 1, f(x) \geq f(y)$ if $x > y$). The weighting vector $\omega(t)$ is defined as

$$\omega^{t_p} = f\left(\frac{p}{P}\right) - f\left(\frac{p-1}{P}\right), p = 1, 2, \ldots, P$$

where the sequence $\{\omega^{t_p}\}$ is a monotonic increasing (or decreasing) sequence. According to Xu (2012) we define this function as:

$$\omega^{t_p} = \frac{e^{\frac{\alpha p}{P}} - 1}{e^{\alpha} - 1} - \frac{e^{\frac{\alpha(p-1)}{P}} - 1}{e^{\alpha} - 1} = \frac{e^{\frac{\alpha p}{P}} - e^{\frac{\alpha(p-1)}{P}}}{e^{\alpha} - 1}$$

The three-steps approach used to solve the multi-period multi-attribute decision making problems is the following:

- The first step is to derive the overall attribute values of the alternatives $\mathscr{A}_i$ at the period t_p by using the basic version of the *DLWA*, the linguistic weighted averaging (*LWA*) operator

$$\begin{aligned} z_i(t_p) &= LWA(r_{i1}^{t_p}, r_{i2}^{t_p}, \ldots, r_{im}^{t_p}) \\ &= \omega_1^{t_p} r_{i1}^{t_p} \oplus \omega_2^{t_p} r_{i2}^{t_p} \oplus \ldots \omega_m^{t_p} r_{im}^{t_p}, \end{aligned} \tag{1}$$

  where $p = 1, 2, \ldots, P$, and m stands for the number of attributes such that $\omega_j^{t_p}$ corresponds to the j-th attribute value at time t_p, with $j = 1, 2, \ldots, m$.

- The second step, is to aggregate the overall attributes values $z_i(t_p)$ $(p = 1, 2, \ldots, P)$ collected from P different periods, and get the overall attribute value z_i of the alternative $\mathscr{A}_i$ by using the *DLWA* operator as follows:

$$\begin{aligned} z_i &= DLWA_{\omega(t)}(z_i(t_1), z_i(t_2), \ldots, z_i(t_P)) \\ &= \omega^{t_1} z_i(t_1) \oplus \omega^{t_2} z_i(t_2) \oplus \ldots \omega^{t_P} z_i(t_P), \end{aligned} \quad (2)$$

  with $i = 1, 2, \ldots, n$.

- The third step is to rank the alternatives $\mathscr{A}$ according to the overall attributes values z_i.

We apply MP-MADM as a decision-making supporting tool for investment scenarios. An investor wants to invest 100 dollars in one of the following most profitable cryptocurrency:

- $\mathscr{A}_1$: Ethereum - ETH
- $\mathscr{A}_2$: Bitcoin Cash - BCH
- $\mathscr{A}_3$: Ripple - XRP
- $\mathscr{A}_4$: Bitcoin - BTC
- $\mathscr{A}_5$: Cardano - ADA.

Periodically, the investor have to decide if he/she maintains or change his/her investment. We consider the following relevant features:

- G^1: day-profitability, which is obtained in terms of the ratio given by closing and opening price for a certain cryptocurrency,
- G^2: day-variability, calculated in terms of the ratio highest/lowest price for a given cryptocurrency,
- G^3: (*log* of) Market capitalization, which corresponds to the relative size of the cryptocurrency based on the price and circulating supply, where the latter is the best possible approximation of the number of coins that are circulating in the market and in public wallets.

In this case of study, the attributes G^1, and G^3 are considered as benefit attributes, while G^2 is considered as a cost attribute in order to penalize high fluctuations on the price of the coin, diminishing in some degree the risk perception on the investor. To facilitate the evaluation's process, which is typically performed by humans, each attribute has been modeled as a linguistic variable with five linguistic values or labels: "very bad", "bad", "neutral", "good" and "very good".

Table: Values of the attributes of the cryptocurrencies that are of our interest. The table shows the values from day 0 to day 4.

Cryp.	Attribute					
		t_0	t_1	t_2	t_3	t_4
$\mathscr{A}_1$	G^1	0.89	0.85	1.06	0.9	0.98
	G^2	1.19	1.24	1.07	1.15	1.09
	G^3	24.32	24.21	24.11	24.17	24.04
$\mathscr{A}_2$	G^1	0.92	0.85	1.18	0.89	0.98
	G^2	1.14	1.21	1.19	1.23	1.13
	G^3	23.05	23.03	22.91	23.11	22.92
$\mathscr{A}_3$	G^1	0.91	0.89	1.05	0.94	1.01
	G^2	1.15	1.16	1.06	1.08	1.09
	G^3	22.98	22.89	22.83	22.87	22.81
$\mathscr{A}_4$	G^1	0.93	0.92	1.05	0.92	0.97
	G^2	1.11	1.11	1.06	1.14	1.07
	G^3	25.11	25.05	25.00	25.06	24.97
$\mathscr{A}_5$	G^1	1.15	1.15	1.15	1.15	1.15
	G^2	1.86	1.85	1.86	1.86	1.86
	G^3	0	0	0	0	0

Then, current values in $\mathscr{A}_i$ are normalized into $\hat{\mathscr{A}_i}$ for each attribute considered as benefit by computing:

$$\hat{\mathscr{A}_i} = \frac{\mathscr{A}_i}{\max_i \mathscr{A}_i}$$

$$\hat{\mathscr{A}_i} = \frac{\min_i \mathscr{A}_i}{\mathscr{A}_i}$$

Then, based on the behavior of the cryptocurrencies during the last five days, the investor needs to rank them in order to invest in the best possible alternative for the following sixth day. Thus, in this example, we assume that the investor wants to invest in the alternative with a "very good" *day-profitability*, a "very good" *day variability* (less fluctuation), a "very good" *trading volume*, and a "very good" *market capitalization*. For simplicity, we also assume that each attribute is equally important.
Torres *et al.* introduced the "*at least*" ($\mathscr{L}$) and "*at most*" ($\mathscr{M}$) unary ordering-based modifiers. Given the label s_q associated to the linguistic value LV_p^j (p fixed), the fuzzy sets *"at least* LV_p^j*"* and *"at most* LV_p^j*"* (abbreviated as $\mathscr{L}(s_q)$ and $\mathscr{M}(s_q)$) are defined as follows:

$$\mathscr{L}(s_q)(x) = \sup\{\mu_p^j(y) \text{ such that } y \in \mathscr{X} \text{ and } y \preceq x\}$$
$$\mathscr{M}(s_q)(x) = \sup\{\mu_p^j(y) \text{ such that } y \in \mathscr{X} \text{ and } x \preceq y\}$$

where $\preceq$ is a crisp ordering on $\mathscr{X}$.

In this example, the linguistic decision model (*LDM*) is constructed as follows, where *taFIA* stands for *time-aware fuzzy information aggregation* approach:

$$LDM{:=}taFIA(\mathscr{L}(LV^{day\_profitability}_{very\_good}),\mathscr{M}(LV^{day\_variability}_{very\_good}),\mathscr{L}(LV^{trading\_volume}_{very\_good})),\mathscr{L}(LV^{market\_capitalization}_{very\_good}))),$$

$LV^{attribute}$ represents the linguistic values for such *attribute*, and $\mathscr{L}$ and $\mathscr{M}$ operators as depicted above, such that according to notation introduced throughout this document, $\mathscr{L}(LV^{day\_profitability}_{very\_good}$ means that the selected cryptocurrency should have at least a very good profitability based on the maximization of the ratio $closing\_price/opening\_price$ on a given day. Thus, $\mathscr{M}(LV^{day\_variability}_{very\_good})$ means that selected cryptocurrency should be the one that minimizes price fluctuation on that given day.
The additive linguistic evaluation scale is the following:

$$S_2 = \{s_{-2} = verybad, s_{-1} = bad, s_0 = neutral, s_1 = good, s_2 = verygood\},$$

	$\tilde{\mathscr{A}}_1$	$\tilde{\mathscr{A}}_2$	$\tilde{\mathscr{A}}_3$	$\tilde{\mathscr{A}}_4$	$\tilde{\mathscr{A}}_5$
Day 1	BTC	BCH	XRP	ETH	ADA
Day 2	BTC	XRP	BCH	ETH	ADA
Day 3	BTC	BCH	XRP	ETH	ADA
Day 4	BTC	XRP	BCH	ETH	ADA
Day 5	XRP	BTC	BCH	ETH	ADA

Table: Ranking of best alternatives using DLWA operator

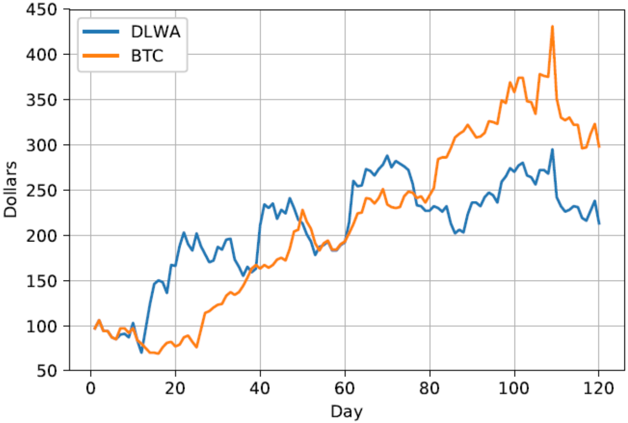

Figure 1: Dollars obtained when selling at closing value from a set of 5 cryptocurrencies

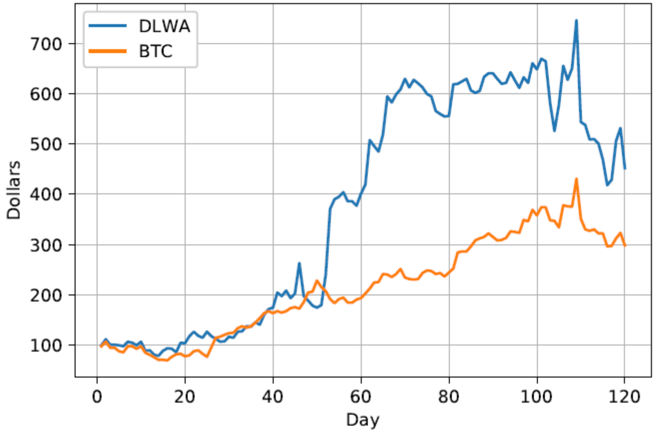

Figure 2: Dollars obtained when selling at closing value from a set of 15 cryptocurrencies

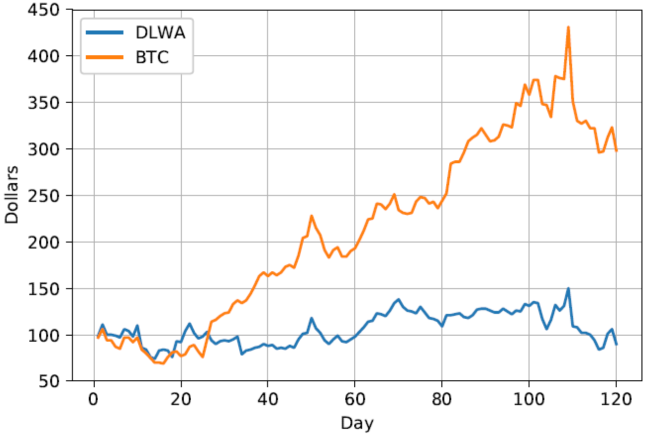

Figure 3: Dollars obtained when selling at closing value from a set of 20 cryptocurrencies

Concluding remarks & open questions:

- Cryptocurrency market is highly volatile and investing on a given digital asset is a complex task, specially without the help of experts assessments, so data-driven approaches arise.
- MP-MADM is a useful tool to support the selection of the best alternative: evaluation and aggregation of several criteria obtained in different times.
- Why including more cryptocurrencies give better results?
- Is it possible to include other assets as well?

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